Preparing for A Level Mathematics

A Level Maths is an extremely rewarding and highly regarded qualification. It does, however, build on skills and knowledge that you have developed at GCSE and the long summer break between the end of Year 11 and the start of Year 12 is often a period when hard-earned skills go rusty. The work below is designed to help you prepare for a flying start in Year 12.

The AS Maths course has three sections; Core 1 and Core 2 build on some of the algebra and trigonometry you have learned, particularly at the A and A* level of GCSE questions, and Statistics builds on the data handling work you have done. The questions below aim to prepare you to start Core 1 and Core 2.

The work is voluntary (although you will receive feedback on it) and you are not required to do it. However, from past experience we know that students who do take a little time to brush up on their skills (once they have had a good break from their exams) can make a very confident start to the course. You can do the work on normal lined paper, and include it into the file of work you will do in Year 12.

Core 1

Indices

1)
State the value of each of the following.

(i) 2^{-3} **[1]**

(ii) 9^0 **[1]**

2)
(i) Simplify $(5a^2b)^3 \times 2b^4$. **[2]**

(ii) Evaluate $\left(\frac{1}{16}\right)^{-1}$. **[1]**

3)
Simplify $\dfrac{(3xy^4)^3}{6x^5y^2}$. **[3]**

4)
(i) Write down the value of $\left(\frac{1}{4}\right)^0$. [1]

(ii) Find the value of $16^{-\frac{3}{2}}$. [3]

5)
Find the value of $\left(\dfrac{1}{2}\right)^{-5}$. [2]

6)
Find the value of each of the following, giving each answer as an integer or fraction as appropriate.

(i) $25^{\frac{3}{2}}$ [2]

(ii) $\left(\dfrac{7}{3}\right)^{-2}$ [2]

7)

(i) Evaluate $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$. **[2]**

(ii) Simplify $\dfrac{(2ac^2)^3 \times 9a^2c}{36a^4c^{12}}$. **[3]**

<u>Surds</u>

1)

(i) Simplify $\dfrac{\sqrt{48}}{2\sqrt{27}}$. **[2]**

(ii) Expand and simplify $(5 - 3\sqrt{2})^2$. **[3]**

2)

(i) Simplify $\sqrt{98} - \sqrt{50}$. [2]

(ii) Express $\dfrac{6\sqrt{5}}{2+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers. [3]

3)

(i) Express $\sqrt{48} + \sqrt{27}$ in the form $a\sqrt{3}$. **[2]**

(ii) Simplify $\dfrac{5\sqrt{2}}{3-\sqrt{2}}$. Give your answer in the form $\dfrac{b + c\sqrt{2}}{d}$. **[3]**

4)

(i) Express $\dfrac{1}{5+\sqrt{3}}$ in the form $\dfrac{a + b\sqrt{3}}{c}$, where a, b and c are integers. [2]

(ii) Expand and simplify $(3 - 2\sqrt{7})^2$. [3]

<u>Algebraic fractions</u>

1)

Factorise $x^2 - 4$ and $x^2 - 5x + 6$.

Hence express $\dfrac{x^2 - 4}{x^2 - 5x + 6}$ as a fraction in its simplest form. [3]

2)

Factorise and hence simplify $\dfrac{3x^2 - 7x + 4}{x^2 - 1}$. [3]

Solving linear inequalities

1)

Solve the inequality $6(x+3) > 2x + 5$. **[3]**

2)

Solve the inequality $3x - 1 > 5 - x$. [2]

3)

Solve the inequality $\frac{5x-3}{2} < x + 5$. **[3]**

4)

Solve the inequality $\frac{3(2x+1)}{4} > -6$. [4]

5)

Solve the inequality $7 - x < 5x - 2$. **[3]**

6)

Solve the inequality $1 - 2x < 4 + 3x$. [3]

Solving equations

1)

Solve the equation $\frac{4x+5}{2x} = -3$. [3]

2)

Solve the equation $\frac{3x+1}{2x} = 4$. **[3]**

3)

Solve the equation $y^2 - 7y + 12 = 0$.

Hence solve the equation $x^4 - 7x^2 + 12 = 0$. [4]

4)

Solve the equation $4x^2 + 20x + 25 = 0$. [2]

Forming and solving equations

1)

The triangle shown in Fig. 10 has height $(x + 1)$ cm and base $(2x - 3)$ cm. Its area is $9\ \text{cm}^2$.

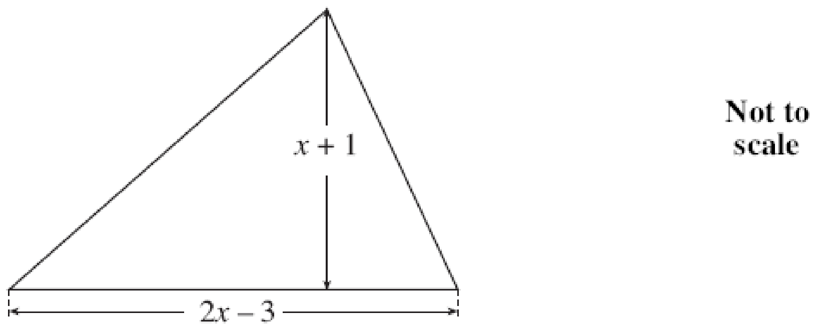


Fig. 10

(i) Show that $2x^2 - x - 21 = 0$. [2]

(ii) By factorising, solve the equation $2x^2 - x - 21 = 0$. Hence find the height and base of the triangle. [3]

2)

Fig. 9 shows a trapezium ABCD, with the lengths in centimetres of three of its sides.

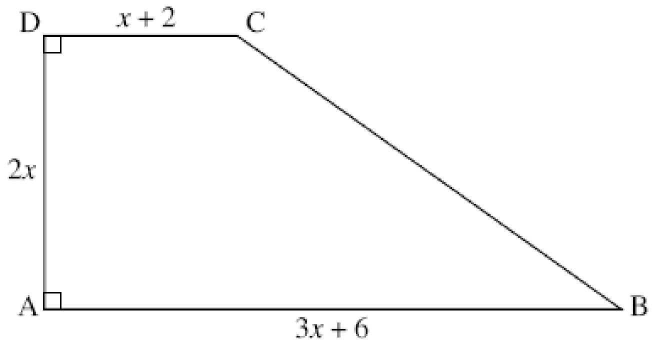


Fig. 9

This trapezium has area $140\ \text{cm}^2$.

(i) Show that $x^2 + 2x - 35 = 0$. **[2]**

(ii) Hence find the length of side AB of the trapezium. **[3]**

Completing the square and turning points

1)

(i) Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$. **[3]**

(ii) Write down the coordinates of the minimum point on the graph of $y = x^2 + 6x + 5$. **[2]**

2)

(i) Write $x^2 - 7x + 6$ in the form $(x - a)^2 + b$. [3]

(ii) State the coordinates of the minimum point on the graph of $y = x^2 - 7x + 6$. [2]

Changing the subject of a formula

1)

Make a the subject of the formula $s = ut + \frac{1}{2}at^2$. **[3]**

2)

The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$. Make r the subject of this formula. [3]

3)

Make y the subject of the formula $a = \frac{\sqrt{y} - 5}{c}$. **[3]**

4)

Rearrange the formula $c = \sqrt{\frac{a + b}{2}}$ to make a the subject. **[3]**

5)

The volume V of a cone with base radius r and slant height l is given by the formula

$$V = \tfrac{1}{3}\pi r^2 \sqrt{l^2 - r^2}.$$

Rearrange this formula to make l the subject. **[4]**

6)

Make a the subject of the equation

$$2a + 5c = af + 7c.$$ [3]

7)

Rearrange $y + 5 = x(y + 2)$ to make y the subject of the formula. **[4]**

8)

Make C the subject of the formula $P = \frac{C}{C + 4}$. [4]

9)

Make x the subject of the formula $y = \frac{1 - 2x}{x + 3}$. **[4]**

Equation of a straight line

1)

Find the equation of the line which is parallel to $y = 5x - 4$ and which passes through the point (2, 13). Give your answer in the form $y = ax + b$. **[3]**

2)

Find the equation of the line which is parallel to $y = 3x + 1$ and which passes through the point with coordinates (4, 5). **[3]**

3)

Find the equation of the line passing through $(-1, -9)$ and $(3, 11)$. Give your answer in the form $y = mx + c$. **[3]**

4)

A line has equation $3x + 2y = 6$. Find the equation of the line parallel to this which passes through the point $(2, 10)$. [3]

5)

(i) Find the equation of the line passing through A $(-1, 1)$ and B $(3, 9)$. [3]

(ii) Show that the equation of the perpendicular bisector of AB is $2y + x = 11$. [4]

Intersection of two lines

1)

Find the coordinates of the point of intersection of the lines $y = 3x + 1$ and $x + 3y = 6$. [3]

2)

Find, algebraically, the coordinates of the point of intersection of the lines $y = 2x - 5$ and $6x + 2y = 7$. [4]

3)

Solve the simultaneous equations $y = x^2 - 6x + 2$ and $y = 2x - 14$.

4)

Find algebraically the coordinates of the points of intersection of the curve $y = 4x^2 + 24x + 31$ and the line $x + y = 10$. **[5]**

5)

Find the coordinates of the points of intersection of the circle $x^2 + y^2 = 25$ and the line $y = 3x$. Give your answers in surd form. [5]

6)

A circle has equation $x^2 + y^2 = 45$.

(i) State the centre and radius of this circle. [2]

(ii) The circle intersects the line with equation $x + y = 3$ at two points, A and B. Find algebraically the coordinates of A and B.

Show that the distance AB is $\sqrt{162}$. [8]

Core 1 and Core 2

Transformation of graphs

1)

The point P (5, 4) is on the curve $y = f(x)$. State the coordinates of the image of P when the graph of $y = f(x)$ is transformed to the graph of

(i) $y = f(x - 5)$, **[2]**

(ii) $y = f(x) + 7$. **[2]**

2)

The curve $y = f(x)$ has a minimum point at (3, 5).

State the coordinates of the corresponding minimum point on the graph of

(i) $y = 3f(x)$, **[2]**

(ii) $y = f(2x)$. **[2]**

3)

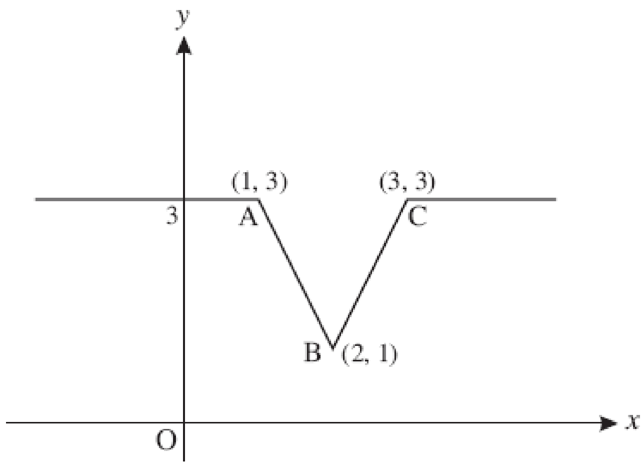


Fig. 4

Fig. 4 shows a sketch of the graph of $y = \mathrm{f}(x)$. On separate diagrams, sketch the graphs of the following, showing clearly the coordinates of the points corresponding to A, B and C.

(i) $y = 2\mathrm{f}(x)$ [2]

(ii) $y = \mathrm{f}(x + 3)$ [2]

4)

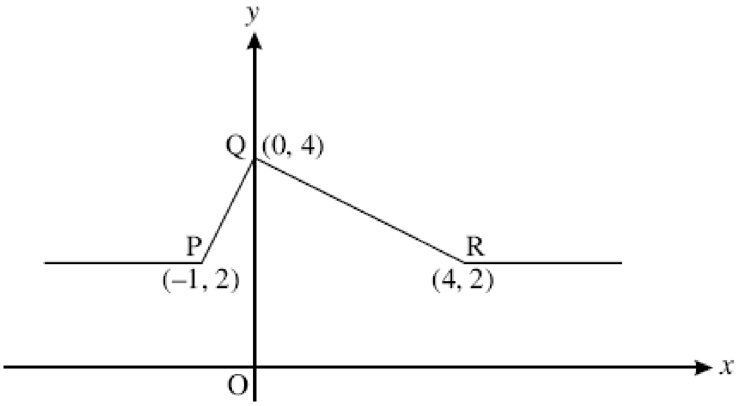


Fig. 5

Fig. 5 shows a sketch of the graph of $y = \mathrm{f}(x)$. On separate diagrams, sketch the graphs of the following, showing clearly the coordinates of the points corresponding to P, Q and R.

(i) $y = \mathrm{f}(2x)$ **[2]**

(ii) $y = \frac{1}{4}\mathrm{f}(x)$ **[2]**

5)

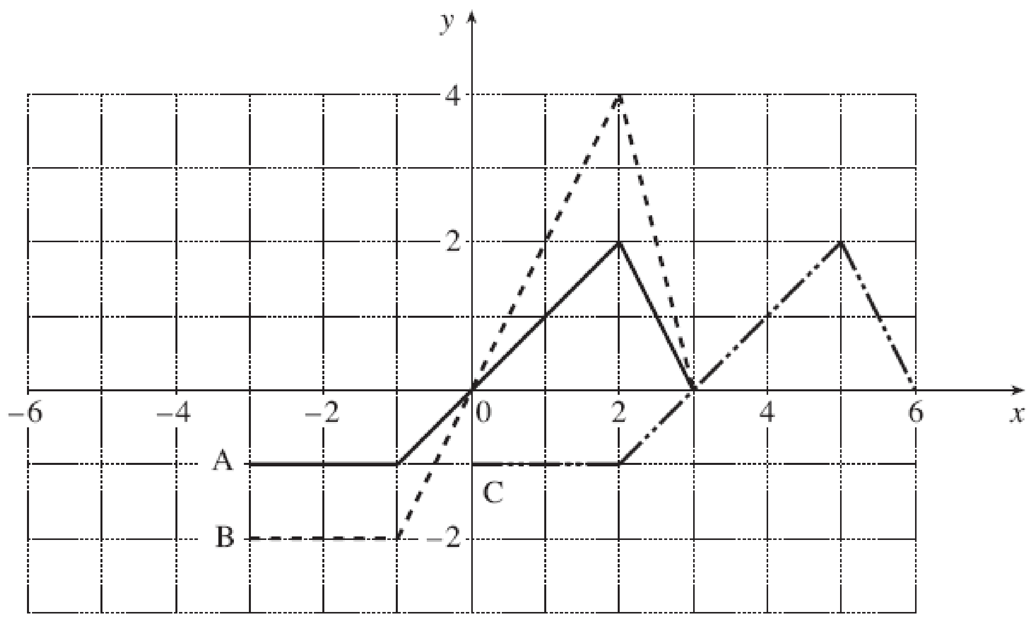


Fig. 3

Fig. 3 shows sketches of three graphs, A, B and C. The equation of graph A is $y = \mathrm{f}(x)$.

State the equation of

(i) graph B, [2]

(ii) graph C. [2]

Core 2

Trigonometric graphs

1)

Sketch the curve $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

2)

Sketch the graph of $y = \cos x$ for $0° \leqslant x \leqslant 360°$.

On the same axes, sketch the graph of $y = \cos 2x$ for $0° \leqslant x \leqslant 360°$. Label each graph clearly. [3]

Sine rule and cosine rule

1)

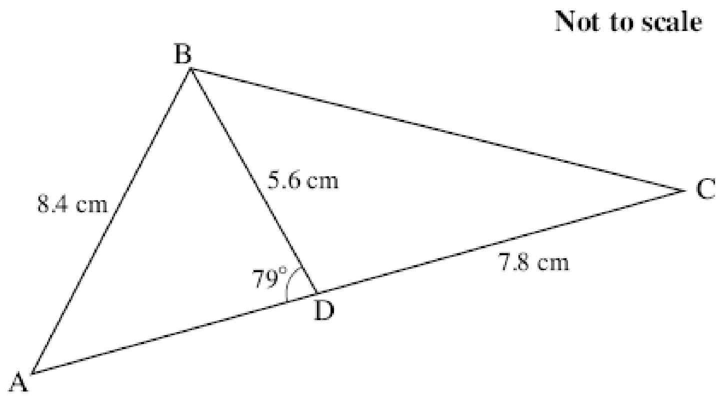


Fig. 7

Fig. 7 shows triangle ABC, with AB = 8.4 cm. D is a point on AC such that angle ADB = 79°, BD = 5.6 cm and CD = 7.8 cm.

Calculate

(i) angle BAD, **[2]**

(ii) the length BC. **[3]**

2)

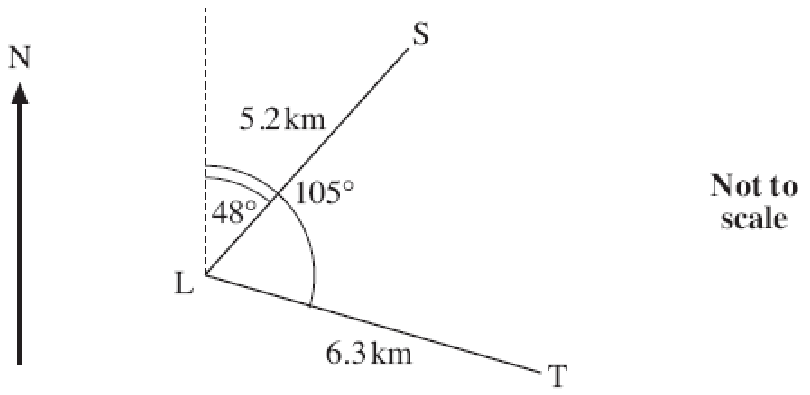


Fig. 10.1

At a certain time, ship S is 5.2 km from lighthouse L on a bearing of 048°. At the same time, ship T is 6.3 km from L on a bearing of 105°, as shown in Fig. 10.1.

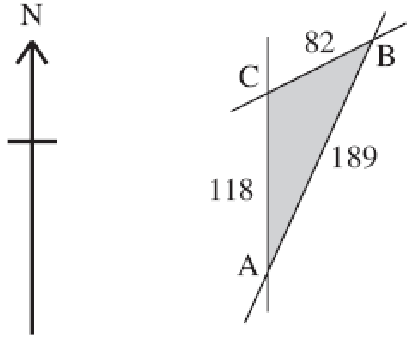
For these positions, calculate

(*A*) the distance between ships S and T, [3]

(*B*) the bearing of S from T. [3]

3)

Fig. 11.1 shows a village green which is bordered by 3 straight roads AB, BC and CA. The road AC runs due North and the measurements shown are in metres.



Not to scale

Fig. 11.1

(i) Calculate the bearing of B from C, giving your answer to the nearest 0.1°. [4]

(ii) Calculate the area of the village green. [2]

Arc length and sector area

1)

A sector of a circle of radius 18.0 cm has arc length 43.2 cm.

Find the angle of the sector. **[2]**

2)

A sector of a circle of radius 5 cm has area 9 cm^2.

Find the perimeter of the sector. [5]